5. Real-time models: Timed Automata

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https://fm-dcc.github.io/sv2425

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CISTER - Research Centre in
Real-Time & Embedded
Computing Systems

Motivation

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Specifying an airbag saying that in a car crash the airbag eventually inflates maybe not enough, but:

in a car crash the airbag eventually inflates within 20ms

> *Correctness in time-critical systems not only depends on the logical result of the computation, but also on the time at which the results are produced*
>
> [Baier & Katoen, 2008]

Examples of time-critical systems

Lip-synchronization protocol

Synchronizes the separate video and audio sources bounding on the amount of time mediating the presentation of a video frame and the corresponding audio frame. Humans tolerate less than 160 ms.

Bounded retransmission protocol

Controls communication of large files over infrared channel between a remote control unit and a video/audio equipment. Correctness depends crucially on

- transmission and synchronization delays
- time-out values for times at sender and receiver

And many others...

- medical instruments
- hybrid systems (eg for controlling industrial plants)

Course structure

1. **Real-time models**
   - Timed Automata and Hybrid Automata
   - Temporal logic
   - Static verification using UPPAAL
2. **Program verification**
   - First Order Logic revisited
   - Abstract Program Semantics
   - Design by Contract and Hoare Logic
   - Verification of annotated programs
3. **Requirements**
   - SAT and SMT solvers
   - *Automatic* theorem proving using Z3
   - Introduction to *Interactive* theorem proving using Coq

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Motivation

- timed transition systems, timed Petri nets, timed IO automata, timed process algebras and other formalisms associate lower and upper bounds to transitions, but no time constraints to transverse the automaton.
- Expressive power is often somehow limited and infinite-state LTS (introduced to express dense time models) are difficult to handle in practice

Motivation

Example

Typical process algebra tools are unable to express a system which has one action *a* which can only occur at time point 5 with the effect of moving the system to its initial state.

This example has, however, a simple description in terms of time measured by a stopwatch:

1. Set the stopwatch to 0
2. When the stopwatch measures 5, action *a* can occur. If *a* occurs go to 1., if not idle forever.

Motivation

This suggests resorting to an automaton-based formalism with an explicit notion of clock (stopwatch) to control availability of transitions.

Timed Automata [Alur & Dill, 90]

- emphasis on decidability of the reachability problem and corresponding practically efficient algorithms
- infinite underlying timed transition systems are converted to finitely large symbolic transition systems where reachability becomes decidable (region or zone graphs)

Associated tools

- UPPAAL [Behrmann, David, Larsen, 04]
- IMITATOR [André, 09]
- PRISM [Parker, Kwiatkowska, 00]
- KRONOS [Bozga, 98]

Motivation

UPPAAL = (Uppsala University + Aalborg University) [1995]

- A toolbox for modeling, simulation and verification of real-time systems
- where systems are modeled as networks of timed automata enriched with integer variables, structured data types, channel syncronisations and urgency annotations
- Properties are specified in a subset of CTL

`www.uppaal.org`

Timed Automata

Timed automata

Finite-state machine equipped with a finite set of real-valued clock variables (clocks)

Clocks

- dense-time model
- clocks can only be inspected or
- reset to zero, after which they start increasing their value implicitly as time progresses
- the value of a clock corresponds to time elapsed since its last reset
- all clocks proceed synchronously (at the same rate)

Timed automata Definition

$$\langle \mathbf{L}, \mathbf{L_0}, \mathbf{Act}, \mathbf{C}, \mathbf{Tr}, \mathbf{Inv} \rangle$$

where

- L is a set of locations, and $L_0 \subseteq L$ the set of initial locations
- Act is a set of actions and C a set of clocks
- $Tr \subseteq L \times \mathcal{C}(C) \times Act \times \mathcal{P}(C) \times L$ is the transition relation

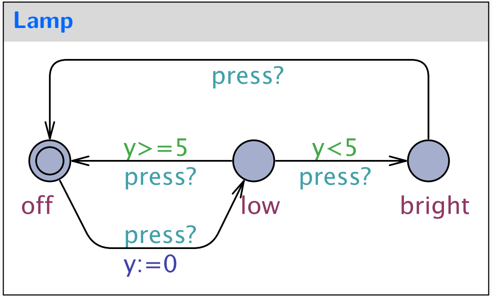
  $$\ell_1 \xrightarrow{g,a,U} \ell_2$$

  denotes a transition from location ℓ_1 to ℓ_2, labelled by a, enabled if guard g is valid, which, when performed, resets the set U of clocks
- $Inv : L \longrightarrow \mathcal{C}(C)$ is the assignment of invariants to locations

where $\mathcal{C}(C)$ denotes the set of clock constraints over a set C of clock variables

Example: the lamp interrupt

(extracted from UPPAAL)



Ex. 5.1: Define $\langle L, L_0, Act, C, Tr, Inv \rangle$.

Clock constraints

$\mathcal{C}(C)$ denotes the set of clock constraints over a set C of clock variables. Each constraint is formed according to

$$g \;::=\; x \,\Box\, n \;\mid\; x - y \,\Box\, n \;\mid\; g \wedge g \;\mid\; \mathit{true}$$

where $x, y \in C$, $n \in \mathbb{N}$ and $\Box \in \{<, \leq, >, \geq, =\}$

used in

- transitions as guards (enabling conditions)

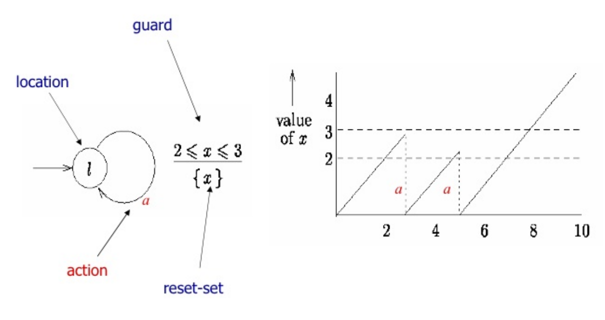
  a transition cannot occur if its guard is invalid

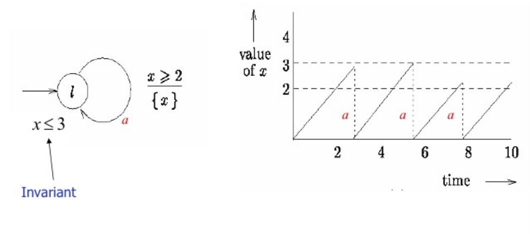
- locations as invariants (safety specifications)

  a location must be left before its invariant becomes invalid

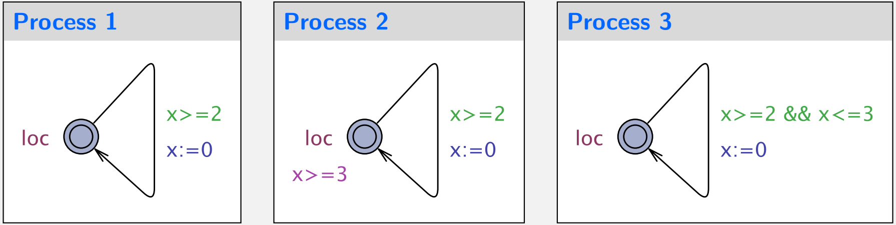
Note

Invariants are the only way to force transitions to occur


guard
location
$2 \leqslant x \leqslant 3$
$\{x\}$
l
a
action
reset-set
value
of x
4
3
2
a
a
2
4
6
8
10


l
$x \geqslant 2$
$\{x\}$
a
$x \leq 3$
Invariant
value
of x
4
3
2
a
a
a
2
4
6
8
10
time

Demo (in Uppaal)



Parallel composition of timed automata

- Action labels as channel identifiers
- Communication by forced handshacking over a subset of common actions
- Is defined as an automaton construction over a finite set of timed automata originating a so-called network of timed automata

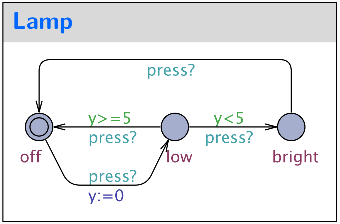
Parallel composition of timed automata

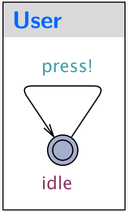
Let $H \subseteq Act_1 \cap Act_2$. The parallel composition of ta_1 and ta_2 synchronizing on H is the timed automata

$$ta_1 \parallel_H ta_2 := \langle L_1 \times L_2, L_{0,1} \times L_{0,2}, Act_{\parallel_H}, C_1 \cup C_2, Tr_{\parallel_H}, Inv_{\parallel_H} \rangle$$

where

- $Act_{\parallel_H} = ((Act_1 \cup Act_2) - H) \cup \{\tau\}$
- $Inv_{\parallel_H}\langle \ell_1, \ell_2 \rangle = Inv_1(\ell_1) \wedge Inv_2(\ell_2)$
- $Tr_{\parallel_H}$ is given by:
  - $\langle \ell_1, \ell_2 \rangle \xrightarrow{g,a,U} \langle \ell_1', \ell_2 \rangle$ if $a \notin H \wedge \ell_1 \xrightarrow{g,a,U} \ell_1'$
  - $\langle \ell_1, \ell_2 \rangle \xrightarrow{g,a,U} \langle \ell_1, \ell_2' \rangle$ if $a \notin H \wedge \ell_2 \xrightarrow{g,a,U} \ell_2'$
  - $\langle \ell_1, \ell_2 \rangle \xrightarrow{g,\tau,U} \langle \ell_1', \ell_2' \rangle$ if $a \in H \wedge \ell_1 \xrightarrow{g_1,a,U_1} \ell_1' \wedge \ell_2 \xrightarrow{g_2,a,U_2} \ell_2'$
    with $g = g_1 \wedge g_2$ and $U = U_1 \cup U_2$



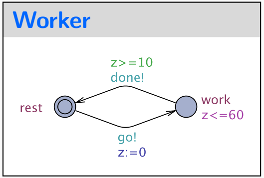


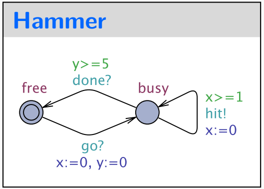
Uppaal:

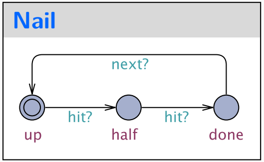
- takes $H = Act_1 \cap Act_2$ (actually as complementary actions denoted by the ? and ! annotations)
- only deals with closed systems

Ex. 5.2: Define the TA of the composition.

Exercise: worker, hammer, nail







Ex. 5.3: Define the TA of the composition.

Semantics

Timed Labelled Transition Systems

Syntax	Semantics
How to write	*How to execute*
Process Languages	LTS (Labelled Transition Systems)
Timed Automaton	TLTS (Timed LTS)

Timed Labelled Transition Systems

Syntax	Semantics
How to write	*How to execute*
Process Languages	LTS (Labelled Transition Systems)
Timed Automaton	TLTS (Timed LTS)

Timed LTS

Introduce delay transitions to capture the passage of time within a LTS:

$$s \xrightarrow{a} s' \quad \text{for } a \in Act\text{, are ordinary transitions due to action occurrence}$$

$$s \xrightarrow{d} s' \quad \text{for } d \in R_0^+\text{, are delay transitions}$$

subject to a number of constraints, eg,

Dealing with time in system models

Timed LTS

- time additivity

$$(s \xrightarrow{d} s' \wedge 0 \leq d' \leq d) \Rightarrow s \xrightarrow{d'} s'' \xrightarrow{d-d'} s' \text{ for some state } s''$$

- delay transitions are deterministic

$$(s \xrightarrow{d} s' \wedge s \xrightarrow{d} s'') \Rightarrow s' = s''$$

Semantics of Timed Automata

Semantics of TA:

Every TA *ta* defines a TLTS

$$\mathcal{T}(ta)$$

whose states are pairs

$$\langle \text{location}, \text{clock valuation} \rangle$$

with infinitely, even uncountably many states, and infinite branching

Clock valuations

Definition

A clock valuation η for a set of clocks C is a function

$$\eta : C \longrightarrow \mathcal{R}_0^+$$

assigning to each clock $x \in C$ its current value $\eta\, x$.

Satisfaction of clock constraints

$$\eta \models x \,\square\, n \;\Leftrightarrow\; \eta\, x \,\square\, n$$
$$\eta \models x - y \,\square\, n \;\Leftrightarrow\; (\eta\, x - \eta\, y) \,\square\, n$$
$$\eta \models g_1 \wedge g_2 \;\Leftrightarrow\; \eta \models g_1 \wedge \eta \models g_2$$

Operations on clock valuations

Delay

For each $d \in \mathcal{R}_0^+$, valuation $\eta + d$ is given by

$$(\eta + d)\,x \;=\; \eta\,x \;+\; d$$

Reset

For each $R \subseteq C$, valuation $\eta[R]$ is given by

$$\begin{cases} \eta[R]\,x \;=\; \eta\,x & \Leftarrow x \notin R \\ \eta[R]\,x \;=\; 0 & \Leftarrow x \in R \end{cases}$$

Let $ta = \langle L, L_0, Act, C, Tr, Inv\rangle$

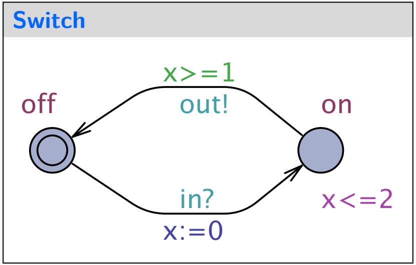
$$\mathcal{T}(ta) \;=\; \langle S, S_0 \subseteq S, N, T\rangle$$

where

- $S = \{\langle l, \eta\rangle \in L \times (\mathcal{R}_0^+)^C \mid \eta \models Inv(l)\}$
- $S_0 = \{\langle \ell_0, \eta\rangle \mid \ell_0 \in L_0 \;\wedge\; \eta\, x = 0 \text{ for all } x \in C\}$
- $N = Act \cup \mathcal{R}_0^+$ (ie, transitions can be labelled by actions or delays)
- $T \subseteq S \times N \times S$ is given by:

$$\langle l, \eta\rangle \xrightarrow{a} \langle l', \eta'\rangle \quad \Leftarrow \quad \exists_{l \xrightarrow{g,a,U} l' \in Tr} \;\; \eta \models g \;\wedge\; \eta' = \eta[U] \;\wedge\; \eta' \models Inv(l')$$

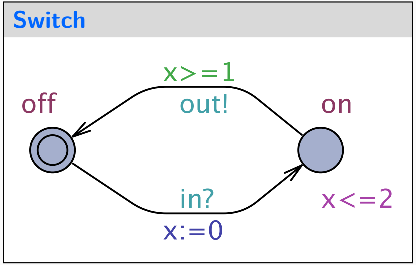
$$\langle l, \eta\rangle \xrightarrow{d} \langle l, \eta + d\rangle \quad \Leftarrow \quad \exists_{d \in \mathcal{R}_0^+} \;\; \eta + d \models Inv(l)$$



Ex. 5.4: Define $\mathcal{T}(\text{SwitchA})$

$$S =$$

Example: the simple switch

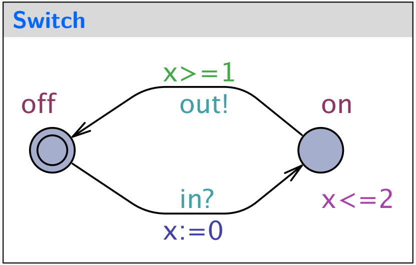


Ex. 5.4: Define $\mathcal{T}$(SwitchA)

$$S = \{\langle off, \overline{t}\rangle \mid t \in \mathcal{R}_0^+\} \cup \{\langle on, \overline{t}\rangle \mid 0 \leq t \leq 2\}$$

where $\overline{t}$ is a shorthand for η such that $\eta\, x = t$

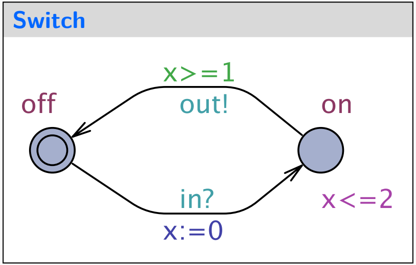
Example: the simple switch



Ex. 5.4: Define $\mathcal{T}(\text{SwitchA})$

$$T = \ldots$$

Example: the simple switch



Ex. 5.4: Define $\mathcal{T}$(SwitchA)

$$\langle off, \overline{t}\rangle \xrightarrow{d} \langle off, \overline{t}+d\rangle \text{ for all } t, d \geq 0$$

$$\langle off, \overline{t}\rangle \xrightarrow{in} \langle on, \overline{0}\rangle \text{ for all } t \geq 0$$

$$\langle on, \overline{t}\rangle \xrightarrow{d} \langle on, \overline{t}+d\rangle \text{ for all } t, d \geq 0 \text{ and } t+d \leq 2$$

$$\langle on, \overline{t}\rangle \xrightarrow{out} \langle off, \overline{t}\rangle \text{ for all } 1 \leq t \leq 2$$

Note

- The elapse of time in timed automata only takes place in locations:
- … actions take place instantaneously
- Thus, several actions may take place at a single time unit

Behaviours

- Paths in $\mathcal{T}(ta)$ are discrete representations of continuous-time behaviours in ta
- ... at least they indicate the states immediately before and after the execution of an action
- However, as interval delays may be realised in uncountably many different ways, different paths may represent the same behaviour

Behaviours

- Paths in $\mathcal{T}(ta)$ are discrete representations of continuous-time behaviours in ta
- ... at least they indicate the states immediately before and after the execution of an action
- However, as interval delays may be realised in uncountably many different ways, different paths may represent the same behaviour
- ... but not all paths correspond to valid (realistic) behaviours:

undesirable paths:

- time-convergent paths
- timelock paths
- zeno paths

Time-convergent paths

$$\langle l, \eta \rangle \xrightarrow{d_1} \langle l, \eta + d_1 \rangle \xrightarrow{d_2} \langle l, \eta + d_1 + d_2 \rangle \xrightarrow{d_3} \langle l, \eta + d_1 + d_2 + d_3 \rangle \xrightarrow{d_4} \cdots$$

such that

$$\forall_{i \in N}.\ d_i > 0 \ \wedge \sum_{i \in N} d_i = d$$

ie, the infinite sequence of delays converges toward d

- Time-convergent path are conterintuitive; as their existence cannot be avoided, they are simply ignored in the semantics of Timed Automata
- Time-divergent paths are the ones in which time always progresses

Time-convergent paths

Definition

An infinite path fragment ρ is time-divergent if $\text{ExecTime}(\rho) = \infty$

Otherwise is time-convergent.

where

$$\text{ExecTime}(\rho) = \sum_{i=0..\infty} \text{ExecTime}(\delta)$$

$$\text{ExecTime}(\delta) = \begin{cases} 0 & \Leftarrow \delta \in Act \\ \delta & \Leftarrow \delta \in \mathcal{R}_0^+ \end{cases}$$

for ρ a path and δ a label in $\mathcal{T}(ta)$

Timelock paths

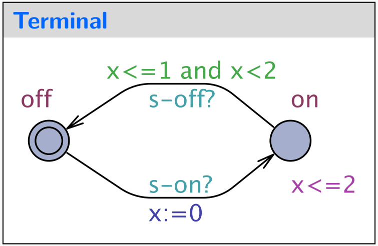
Definition

A path is timelock if it contains a state with a timelock, ie, a state from which there is not any time-divergent path

A timelock represents a situation that causes time progress to halt (e.g. when it is impossible to leave a location before its invariant becomes invalid)

- any teminal state ($\neq$ terminal location) in $\mathcal{T}(ta)$ contains a timelock
- ... but not all timelocks arise as terminal states in $\mathcal{T}(ta)$

Timelock paths

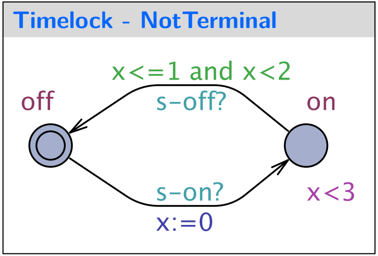


State $\langle on, 2\rangle$ is reachable through path

$$\langle off, 0\rangle \xrightarrow{s-on} \langle on, 0\rangle \xrightarrow{2} \langle on, 2\rangle$$

and is terminal

Timelock paths



State $\langle on, 2\rangle$ is not terminal but has a convergent path:

$$\langle on, 2\rangle\langle on, 2.9\rangle\langle on, 2.99\rangle\langle on, 2.999\rangle\ldots$$

Zeno

In a Timed Automaton

- The elapse of time only takes place at locations
- Actions occur instantaneously: at a single time instant several actions may take place

... it may perform infinitely many actions in a finite time interval
(non realizable because it would require infinitely fast processors)

Zeno

In a Timed Automaton

- The elapse of time only takes place at locations
- Actions occur instantaneously: at a single time instant several actions may take place

... it may perform infinitely many actions in a finite time interval
(non realizable because it would require infinitely fast processors)

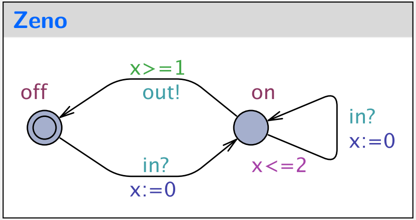
Definition

An infinite path fragment ρ is zeno if it is time-convergent and infinitely many actions occur along it

A timed automaton *ta* is non-zeno if there is not an initial zeno path in $\mathcal{T}(ta)$

Example

Suppose the user can press the *in* button when the light is *on* in



In doing so clock x is reset to 0 and light stays *on* for more 2 time units (unless the button is pushed again ...)

Example

Typical paths: The user presses *in* infinitely fast:

$$\langle off, 0\rangle \xrightarrow{in} \langle on, 0\rangle \xrightarrow{in} \langle on, 0\rangle \xrightarrow{in} \langle on, 0\rangle \xrightarrow{in} \langle on, 0\rangle \xrightarrow{in} \cdots$$

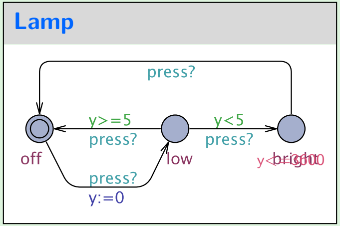
The user presses *in* faster and faster:

$$\langle off, 0\rangle \xrightarrow{in} \langle on, 0\rangle \xrightarrow{0.5} \langle on, 0.5\rangle \xrightarrow{in} \langle on, 0\rangle \xrightarrow{0.25} \langle on, 0.25\rangle \xrightarrow{in} \langle on, 0\rangle \xrightarrow{0.125} \cdots$$

How can this be fixed?

Time shall pass!

Ex. 5.5: Recall our lamp



1. Describe a time-divergent path, if it exists.
2. Describe a time-convergent path, if it exists.
3. Describe a timelock path, if it exists.
4. Is this automata non-zeno? Justify.

Zeno

Sufficient criterion for nonzenoness

A timed automaton is nonzeno if on any of its control cycles time advances with at least some constant amount (≥ 0). Formally, if for every control cycle

$$\ell_0 \xrightarrow{g_0, a_0, U_0} \ell_1 \xrightarrow{g_1, a_1, U_1} \dots \xrightarrow{g_n, a_n, U_n} \ell_0$$

there exists a clock $x \in C$ such that

1. $x \in U_i$ (for $0 \leq i \leq n$)
2. for all clock valuations η, there is a $c \in \mathbb{N}_{>0}$ such that

$$\eta(x) < c \;\Rightarrow\; ((\eta \not\models g_j) \;\vee\; \neg Inv(\ell_j)) \text{ for some } 0 \leq j \leq n$$

Warning

Both

- timelocks
- zenoness

are modelling flaws and need to be avoided.

Example

In the example above, it is enough to impose a non zero minimal delay between successive button pushings.

Modelling in Uppaal

Uppaal

... an editor, simulator and model-checker for TA with extensions ...

Editor.

- Templates and instantiations
- Global and local declarations
- System definition

Simulator.

- Viewers: automata animator and message sequence chart
- Control (eg, trace management)
- Variable view: shows values of the integer variables and the clock constraints defining symbolic states

Verifier.

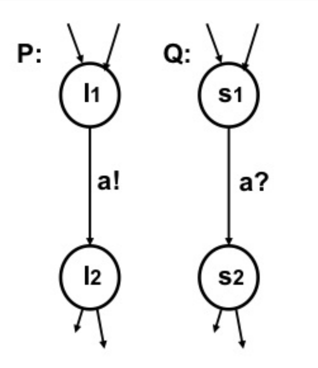
- `(see next session)`

Extensions (modelling view)

- templates with parameters and an instantiation mechanism
- data expressions over bounded integer variables (eg, `int[2..45] x`) allowed in guards, assigments and invariants
- rich set of operators over integer and booleans, including bitwise operations, arrays, initializers ... in general a whole subset of C is available
- non-standard types of synchronization
- non-standard types of locations

Extension: broadcast synchronization

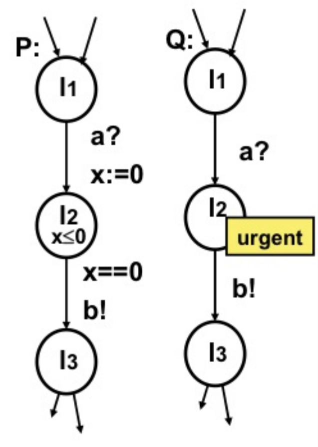
- A sender can synchronize with an arbitrary number of receivers
- Any receiver than can synchronize in the current state must do so
- Broadcast sending is never blocking (the send action can occur even with no receivers).



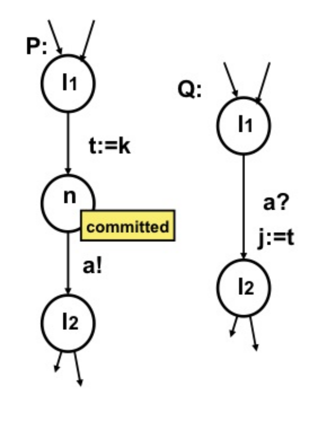
Channel a is declared `urgent chan a` if both edges are to be taken as soon as they are ready (simultaneously in locations ℓ_1 and s_1).

Note the problem can not be solved with invariants because locations ℓ_1 and s_1 can be reached at different moments

- No delay allowed if a synchronization transition on an urgent channel is enabled
- Edges using urgent channels for synchronization cannot have time constraints (ie, clock guards)

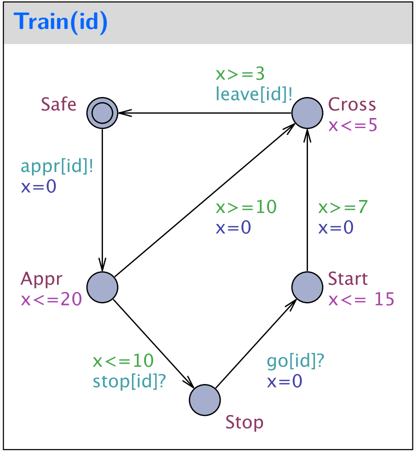


- Time does not progress but interleaving with normal location is allowed
- Both models are equivalent: no delay at an urgent location
- but the use of urgent location reduces the number of clocks in a model and simplifies analysis



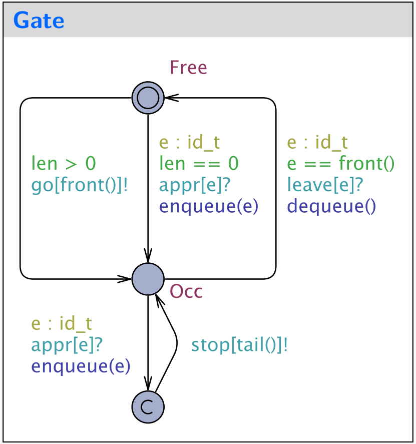
- delay is not allowed and the committed transition must be left in the next instant (or one of them if there are several), i.e., next transition must involve an outgoing edge of at least one of the committed locations
- Our aim is to pass the value k to variable j (via global variable t)
- Location n is committed to ensure that no other automata can assign j before the assignment $j := t$

The train-gate example



- Events model approach/leave, order to stop/go
- A train cannot be stopped or restart instantly
- After approaching it has 10m to receive a stop.
- After that it takes further 10m to reach the cross
- After restarting takes 7 to 15m to reach the cross and 3-5m to cross

The train-gate example



- Note the use of parameters and the select clause on transitions
- Programming