

Concurrent Programming - Exercises 5

Weak bisimilarity and observable congruence

1. Let

$$\begin{aligned} Buffer &:= put?.get?.Buffer \\ BufferL &:= put?.pass!.BufferL \\ BufferR &:= pass?.get?.BufferR \end{aligned}$$

Show $(BufferL|BufferR)\setminus\{pass!,pass?\} \approx (Buffer|Buffer)$.

2. Given the weak bisimilarity \approx

$$\approx = \bigcup_{R \text{ weak bisimulation}} R$$

Show that

- (a) \approx is an equivalence.
- (b) \approx is the largest weak bisimulation.
- (c) The weak bisimilarity is coarsest than the strong bisimilarity, i.e., $\sim \subseteq \approx$.

3. Let

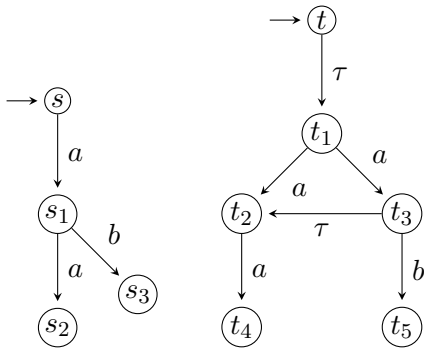
$$\begin{aligned} CMB &:= coin?.coffee!.CMB + coin?.CMB \\ CS &:= pub!.coin!.coffee?.CS \\ UniBad &:= (CMB|CS)\setminus\{coin,coffee\} \end{aligned}$$

Is true that $Spec \approx Unibad$ where $Spec := pub!.Spec$?

4. Show that for any $P, Q \in CCS$ and $\alpha \in Act$ the following equivalences hold:

$$\begin{aligned} \alpha.\tau.P &\approx \alpha.P \\ P + \tau.P &\approx \tau.P \\ \alpha.(P + \tau.Q) &\approx \alpha.(P + \tau.Q) + \alpha.Q \end{aligned}$$

5. Consider the following LTS and show that $s \not\approx t$ using a weak bisimulation game.



6. Consider the following protocol $Protocol := acc?.del!.Protocol$ that corresponds to a communication channel. Given the implementation

$$\begin{aligned}
 Send &:= acc?.Sending \\
 Sending &:= send!.Wait \\
 Wait &:= ack?.Send + error?.Sending \\
 Rec &:= trans?.Del \\
 Del &:= del!.Ack \\
 Ack &:= ack!.Rec \\
 Med &:= send?.Med1 \\
 Med1 &:= \tau.Err + trans!.Med \\
 Err &:= error!.Med
 \end{aligned}$$

show that $(Send|Med|Rec) \setminus \{send, error, trans, ack\} \approx Protocol$. Check with Pseuco.Com.

7. Show that $a.0 + 0 \not\approx a.0 + \tau.0$ and conclude that \approx is not a congruence in CCS .
8. Let \simeq be the observable congruence; show that:
- $\tau.a \not\approx a$
 - $P|\tau.Q \not\approx \tau.(P|Q)$
9. Consider the following algorithms $MUTEX$ for mutual exclusivity. For each one:
- (a) Write it in CCS. Use $enter_i$ and $exits_i$ for $i = 1, 2$ to indicate the critical region.
 - (b) Check if the mutual exclusion seems to be working by randomly navigating through pseuco.com.
 - (c) Implement them also in CCS-CAOS.
 - (d) Consider now:

$$MutexSpec := enter1.exit1.MutexSpec + enter2.exit2.MutexSpec$$

Is true that $MUTEX \approx MutexSpec$?

- i) Peterson algorithm.

For process P_i , $j, i = 1, 2$ and $i \neq j$.

```

while true do
  noncritical actions
   $b_i \leftarrow \mathbf{true};$ 
   $k \leftarrow j;$ 
  while  $b_j \wedge k = j$  do
    skip;
  end while
  critical actions
   $b_i \leftarrow \mathbf{false}$ 
end while

```

- ii) Hyman algorithm. Variables b_i are Boolean and k is an integer. For processes $P_i, j, i = 1, 2$ and $i \neq j$.

```

while true do
  noncritical actions
   $b_i \leftarrow \text{true};$ 
  while  $k \neq i$  do
    while  $b_j$  do
      skip;
    end while
     $k \leftarrow i;$ 
  end while
  critical actions;
   $b_i \leftarrow \text{false};$ 
end while

```

- iii) Pnueli algorithm.

Variables y_i are Boolean and start in *false* being local. The variable s is shared and has value 0 or 1 starting in 1. For process P_i and $i = 0, 1$:

```

while true do
  noncritical actions
   $y_i \leftarrow \text{true};$ 
   $s \leftarrow i;$ 
  while  $\neg(y_{1-i} = 0 \vee (s \neq i))$  do
    skip;
  end while
  critical actions;
   $y_i \leftarrow \text{false};$ 
end while

```

- iv) Simple algorithm.

Variable y is shared and Boolean, and starts in 0. For process P_i and $i = 0, 1$:

```

while true do
  noncritical actions
  while  $y = 0$  do
    skip;
  end while
   $y \leftarrow 0;$ 
  critical actions;
   $y \leftarrow 1;$ 
end while

```

10. Solve the exercises from the Pseuco.com book:

- <https://book.pseuco.com/#/read/equality/weak-bisimilarity>
- <https://book.pseuco.com/#/read/equality/observation-congruence>
- <https://book.pseuco.com/#/read/equality/workout>