Concurrent Programming - Exercises 4

Ismorphism, Trace Equivalence and (Strong) Bisimulation

Consult Chap. 3-1-3,3 e 3.5 of [1].

1. Let

$$\begin{array}{rcl} P & := & a!.0 + b!.0 \\ Q & := & b!.0 + a!.0 \end{array}$$

   determine if the following relations hold

$$\begin{array}{rcl} P & \sim_{tr} & Q \\ a!.P & \sim_{tr} & a!.Q \\ a!.P + a!.P & \sim_{tr} & a!.Q + a!.Q \end{array}$$

   and also for $\sim_{iso}$?

2. Show $(b!.0+c!.0) \sim_{iso} (c!.0+b!.0)$ but $a!.(b!.0+c!0)+a!.(b!.0+c!0) \not\sim_{iso} a!.(b!.0+c!0)+a!.(c!.0+b!.0)$ and conclude that $\sim_{iso}$ is not a congruence.

3. Show that for every $P, Q, R \in CCS$

$$\begin{array}{rcl} P + Q & \sim_{tr} & Q + P \\ (P + Q) + R & \sim_{tr} & P + (Q + R) \\ P + 0 & \sim_{tr} & P, \end{array}$$

   and $\alpha.(P + Q) \sim_{tr} \alpha.P + \alpha.Q$.

4. Let

$$\begin{array}{rcl} CTM & := & coin?.(coffee!.CTM + tea!.CTM) \\ CTM' & := & coin?.coffee!.CTM' + coin?.tea!.CTM' \end{array}$$

   Show that $CTM \sim_{tr} CTM'$.

5. Solve bisimulation problems in `http://tinyurl.com/pseuco`

6. Solve (strong) bisimulation problems in PseuCo Book `https://book.pseuco.com/#/read/equality/workout`

7. Show that
   (a) $\sim$ is an equivalence relation.
   (b) $\sim$ is the largest bisimulation.
   (c) $s \sim t$ iff for each $\alpha \in Act$

- If $s \xrightarrow{\alpha} s'$ then t' such that $t \xrightarrow{\alpha} t'$ e $s' \sim t'$
- If $t \xrightarrow{\alpha} t'$ then s' such that $s \xrightarrow{\alpha} s'$ and $s' \sim t'$.

8. Let $TS = (S, Act, \longrightarrow)$ such that $S = \{s_i \mid i \geq 1\} \cup \{t\}$, $Act = \{a\}$ e $\xrightarrow{a} = \{(s_i, s_{i+1} \mid i \geq 1\} \cup \{(t,t)\}$.

   Show that $s_1 \sim t$ proving that $R = \{(s_i, t) \mid i \geq 1\}$ is a bisimulation.

9. Let P and Q be defined by

$$\begin{aligned} P &:= a.P_1 + b.P_2 \\ P_1 &:= c.P \\ P_2 &:= c.P \end{aligned}$$

   and

$$\begin{aligned} Q &:= a.Q_1 + b.Q_2 \\ Q_1 &:= c.Q_3 \\ Q_2 &:= c.Q_3 \\ Q_3 &:= a.Q_1 + b.Q_2 \end{aligned}$$

   Show that $P \sim Q$ presenting a bisimulation that contains (P, Q). Draw their LTSs and test in pseuCo.com.

10. Let P and Q be defined by

$$\begin{aligned} P &:= a.P_1 \\ P_1 &:= b.P + c.P \end{aligned}$$

   and

$$\begin{aligned} Q &:= a.Q_1 \\ Q_1 &:= b.Q_2 + c.Q \\ Q_2 &:= a.Q_3 \\ Q_3 &:= b.Q + c.Q_2 \end{aligned}$$

   Show that $P \sim Q$ presenting a bisimulation that contains (P, Q).

11. Show that if $P \sim Q$ and $\alpha \in Act$, $R \in CCS$ and $H \subseteq Com$, then

$$\begin{aligned} \alpha.P &\sim \alpha.Q \\ P + R &\sim Q + R \\ R + P &\sim R + Q \\ P|R &\sim Q|R \\ R|P &\sim R|Q \\ P\backslash H &\sim Q\backslash H \end{aligned}$$

12. Let

$$\begin{aligned} P &:= a.(b.0 + c.0) \\ Q &:= a.b.0 + a.c.0 \end{aligned}$$

Show that P and Q are not strongly bisimilar.

13. Show that

$$\begin{aligned} P + Q &\sim Q + P \\ P + 0 &\sim P \\ (P + Q) + R &\sim P + (Q + R) \end{aligned}$$

14. Show that for all $P, Q, R \in CCS$,

$$\begin{aligned} P|Q &\sim Q|P \\ P|0 &\sim P \\ (P|Q)|R &\sim P|(Q|R) \end{aligned}$$

Show that these are bisimulations

$$\begin{aligned} &\{(P|Q, Q|P) \mid P, Q \in CCS\}, \\ &\{(P|0, P) \mid P \in CCS\}, \\ &\{((P|Q)|R, P|(Q|R)) \mid P, Q, R \in CCS\}. \end{aligned}$$

15. Let:

$$\begin{aligned} C_0 &:= inc.C_1 \\ C_n &:= inc.C_{n+1} + dec.C_{n-1}, \text{ para } n \geq 1 \end{aligned}$$

and $C := inc.(C|dec.0)$.

(a) Draw initial fragments of $[\![C]\!]_\Gamma$ and $[\![C_0]\!]_\Gamma$.

(b) Show that the following relation is a bisimulation.

$$\begin{aligned} \mathcal{R} = \; &\{(C|\prod_{i=1}^{k} P_i, C_n) \mid k \geq 0 \wedge (P_i = 0 \vee P_i = dec.0) \\ &\wedge \text{ the number of } i\text{s with } P_i = dec.0 \text{ is } n\} \end{aligned}$$

Consider $(C|\prod_{i=1}^{k} P_i, C_n) \in \mathcal{R}$.
Show that

1. if $C|\prod_{i=1}^{k} P_i \xrightarrow{\alpha} P$ exists Q such that $C_n \xrightarrow{\alpha} Q$ and $(P,Q) \in \mathcal{R}$.
2. if $C_n \xrightarrow{\alpha} Q$ exists P such that $C|\prod_{i=1}^{k} P_i \xrightarrow{\alpha} P$ and $(P,Q) \in \mathcal{R}$.

16. Consider a 1-Buffer.

$$B \;:=\; put?.get?.B$$

For $n \geq 1$ we can consider a Buffer with capacity n, where B_i^n is a buffer with capacity n with $0 \leq i \leq n$ elements.

$$\begin{array}{rcl} B_0^n & := & put?.B_1^n \\ B_i^n & := & put?.B_{i+1}^n + get?.B_{i-1}^n, \;\; 0 < i < n \\ B_n^n & := & get?.B_{n-1}^n \end{array}$$

(a) Verify that $B \sim_{iso} B_0^1$ (draw their LTSs).

(b) Verify that $B_0^2 \sim B_0^1 | B_0^1$ (draw their LTSs).

(c) Show that for $n \geq 1$,

$$B_0^n \sim \underbrace{B_0^1 | B_0^1 | \ldots | B_0^1}_{n}.$$

showing that the following relation is a bisimulation:

$$\mathcal{R} \;=\; \{(B_i^n, B_{i_1}^1 | B_{i_2}^1 | \ldots | B_{i_n}^1) \mid i_j \in \{0,1\} \wedge \sum_{j=1}^{n} i_j = i\}$$

References

[1] L. Aceto, A. Ingólfsdóttir, K.G. Larsen, and J. Srba. *Reactive Systems: Modelling, Specification and Verification.* Cambridge University Press, 2007.